ECHO: SOLUTION TECHNIQUE FOR A NONLINEAR ECONOMIC HARVEST OPTIMIZATION MODEL

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ECHO: -SOLUTION TECHNIQUE FOR A NONLINEAR ECONOMIC HARVEST OPTIMIZATION MODEL

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Abstract.--ECHO extends the Faustmann Formula to forest management and harvesting problems with nonlinear supply and demand conditions. Negatively sloped demand curves and/or positively sloped marginal cost curves can be used to calculate timber harvests which define the optimal time path to a long-run equilibrium forest starting with any initial age class distribution and yield projections. Time horizons, rotation ages, conversion periods, regulated forest conditions and harvest flow constraints are not specified in advance. The solution technique employs an iterative optimum-seeking binary search process which links forest simulation to the necessary and sufficient conditions for maximizing present value.

Additional keywords: Capital theory, financial maturity, forest simulation, forest management, forest regulation, forest rotation ages, allowable cut.

INTRODUCTION

This paper explains an economic model developed by Walker (1971) to optimize the rate of timber harvesting using economic criteria and references work that has subsequently been done or called to the author's attention. Emphasis is given to the solution technique, since some of the major uses and policy implications of this model have been emphasized elsewhere (Walker 1974 and 1975). For brevity and popularized usage, this model has been dubbed with the acronym 'ECHO' taken from economic harvest optimization.

Basically, ECHO is no more than an extension of the Faustmann Formula (Faustmann 1849) which is the earliest known publication that correctly applies discounted cash flow concepts to maximize net present value. The Faustmann Formula has been applied and compared to other financial maturity models by numerous writers. The most definitive work was done by Gaffney (1957) and Samuelson (1974) who most authoritatively verified the work of Faustmann as determining the economically optimal steady-state rotation period for forest trees, or for that matter, the replacement cycle for any appreciating or depreciating capital asset.

In addition to the steady-state conditions, the Faustmann Formula also requires the assumption that the timberland owner faces perfectly elastic factor supply and product demand curves. In other words, to really use the Faustmann Formula, a timberland owner must believe his timber holdings are so small relative to the total markets in which he operates that he cannot

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affect prices. This condition is not met for timberland owners of any substantial size.

The Faustmann Formula is therefore useful only for very small timberland ownerships or for defining the long-run steady-state equilibrium position of larger ownerships. It offers no assistance in defining the optimal time paths to equilibrium positions for timberland owners initially in the disequilibrium positions commonly encountered throughout the industry in both the public and private sectors. The Faustmann Formula may help give rise to the popular misconception that economics without harvest flow constraints implies all mature timber should be harvested immediately or during the first planning period (e.g. Chambers and Pierson 1973 and U.S.F.S. 1973).

The ECHO model and its solution technique can be used by any timberland owner to determine the economically optimal rates of harvest and the most economically attractive set of forest management treatments over time. The objective of ECHO is to maximize the present value of all future net cash flows generated by a timber management program. Inputs to ECHO are the biological yield potential of the forest and the demand and cost functions facing the timberland owner. The biological yield potential of the forest in the form of inventory, growth and yield data is used as the production function or production possibilities schedule of the forest. ECHO does not employ any of the traditional forestry concepts commonly used as constraints on timber harvest. Factors such as community stability, mill requirements or product objectives can be considered through their impact on prices which is expressed by the relevant demand and cost functions. Harvest flow constraints, rotation ages, regulated forest conditions, conversion periods and time horizons are not specified by the user.

CONDITIONS FOR PRESENT VALUE MAXIMIZATION

ECHO utilizes the mathematical structure of a forest biological model and the economic conditions which must be met for present value to be maximized. The forest biological model requires an initial forest inventory and growth and yield projections which can be varied parametrically to find the optimal land base for timber management and the optimal levels of cultural practices over time which will enhance growth. In addition, the forest biological model requires an ordered sequence of harvest priorities. For the simple case first studied by Walker (1971), the forest model has only one site and stocking category with the volume per acre expressed solely as a function of stand age. The annual percentage rate of volume and value growth in merchantable stands is assumed to decline with increasing stand age. Harvest priorities can then be assigned in order of decreasing stand age, since holding the oldest stand will yield the lowest rate of return. A more complex forest model and harvest sequencing criterion will be discussed later.

For the sake of simplicity in developing the mathematical relationships for the ECHO solution technique, time is discretized and the harvests, forest growth and cash flows are treated as though they occur at an instant in time at the beginning of each time period.

The relationship that must exist between the last unit of wood cut at time t, and the first unit cut at time t+1, is expressed by the following first degree difference equation:

$$V_t(MR_t - MC_t)(1+i) = V_{t+1}(MR_{t+1} - MC_{t+1}) \qquad (I)$$

where: V_t = the volume per acre of the last (youngest) age class harvested at time t.

V_{t+1} = the volume per acre of the first (oldest) age class harvested at time t+1.

MR_t = the marginal revenue of the last unit of wood harvested at time t.

MR_{t+1} = the change in total revenue at time t+1 if a unit of wood of the first (oldest) age class is not harvested in t+1.

MC_t = the marginal cost of the last unit of wood harvested at time t.

MC_{t+1} = the change in total cost at time t+1 if a unit of wood of the first (oldest) age class is not harvested in t+1.

i = the rate of interest for equating net cash flows at time t and t+1.

MR_t and MC_t apply to the last unit harvested at time t and therefore fit the normal concepts of marginal revenue and marginal cost. It is important to note that MR_{t+1} and MC_{t+1} apply to a unit of the first age class harvested at time t+1, but they are not the marginal revenue and marginal cost of the first unit. They are the change in total revenue and total cost of including the first unit as part of the total harvest at time t+1. Specifically, this unit is the last one considered for harvest at time t, but left to grow until time t+1. MR_{t+1} and MC_{t+1} will fit the normal concepts of marginal revenue and marginal cost only if a single age class is harvested at time t+1.

Since more than one age class can be harvested at time t+1, MR_{t+1} and MC_{t+1} must be defined in a more general manner. The total revenue at time t+1 is expressed as:

$$TR_{t+1} = P_{t+1}(V_{t+1}^{(j)} \cdot A^{(j)} + V_{t+1}^{(j-1)} \cdot A_{t+1}^{(j-1)} + \ldots\ldots + V_{t+1}^{(j-n+1)} \cdot A^{(j-n+1)}) \qquad (II)$$

where: TR_{t+1} = the total revenue at time t+1.

P_{t+1} = the price of a unit of wood at time t+1.

V = volume per acre.

A = acres.

j = the age of the first (oldest) age class harvested at time t+1.

n = the number of age classes harvested at time t+1.

MR_{t+1} is therefore the change in TR_{t+1} if the volume harvested in the first (oldest) age class $V_{t+1}^{(j)} \cdot A^{(j)}$ were reduced by one unit of wood. MC_{t+1} is defined in an analogous manner.

If the conditions expressed by equation (I) are not met, that is an inequality exists, the present value can be increased by shifting harvest volumes between time t and t+1. Marginal revenues will decrease (increase) as harvest levels increase (decrease) due to negatively sloped demand curves implied by imperfect product markets. Marginal harvesting costs will rise as harvest levels increase if there are imperfect factor markets. If the left-hand side of the equation is greater than the right-hand side, for example, volume must be shifted from time t+1 to time t until equality is obtained to maximize the present value of the net cash flows from these two points in time. Likewise, a shift in harvest volume from time t to t+1 is needed if the right-hand side of the equation is greater than the left-hand side.

The volume growth rate, g, that occurs between times t and t+1 is determined as follows:

$$V_{t+1} = V_t + gV_t \qquad \text{(III)}$$

Rearranging,

$$g = \frac{V_{t+1}}{V_t} - 1 \qquad \text{(IV)}$$

and

$$(1+g) = \frac{V_{t+1}}{V_t} \qquad \text{(V)}$$

Here V_t and V_{t+1} apply to timber of the same age class at time t, because V_{t+1} represents one period of growth added to V_t. An equivalent form of equation (I) is therefore:

$$MR_t - MC_t = \frac{(1+g)(MR_{t+1} - MC_{t+1})}{(1+i)} \qquad \text{(VI)}$$

Consequently, yield equations are not essential for use in ECHO. They can be replaced with individual tree volume tables applied to periodic changes in the forest inventory by diameter classes such as in the TRAS growth model developed by Larson and Goforth (1974).

Likewise, complex forest properties can be analyzed with ECHO by having the user specify an appropriate harvest priorizing rule that ECHO can use to select stands or trees for harvesting as it calculates forward over time. The Western Timber Association (1974) used ECHO on the twenty-nine type-sites of the Stanislaus National Forest by always selecting the stand that would generate the highest net cash flow per unit of timber harvested as the highest logging priority. While there are some instances where this particular rule might preclude present value maximization, this rule is supported by economic theory and in keeping with the general concensus of economists that the rational firm and economy should use superior resources first (Gaffney 1967).

The Kuhn-Tucker theorem (Kuhn and Tucker 1951) demonstrates the necessary and sufficient conditions for solving nonlinear programming problems. Johnson and Scheurman (1974) have shown that the conditions expressed by equation (I) are almost identical to Kuhn-Tucker conditions. The only difference is that equation (I) does not include the equivalent of Lagrange multipliers to account for the costs of delaying future harvests for an additional time period on the land occupied by the last tree considered for harvest at time t, but left to grow until time t+1. Johnson and Scheurman (1974) correctly noted that the manner in which Walker (1971) used equation (I) ignored these land holding costs and obtained a solution that was slightly suboptimal even though he traced harvests out over multiple rotations to a long-run equilibrium. When these land holding costs are ignored, the rate of harvest of the initial inventory is too conservative and the long-run equilibrium harvest is too high.

In the solution technique discussed here, as well as in the quadratic programming approach used by Johnson (1973), it is impossible to precisely calculate the Lagrange multiplier values that will account for land holding costs because of the nonlinear nature of the problem. With ECHO it is rather simple, however, to add a correction coefficient, ϕ, to the interest rate, i, that will closely approximate the correct solutions. This correction coefficient is discussed in detail by Gaffney (1957) for use in obtaining a correct Faustmann Formula harvesting solution by comparing the value growth rate of existing stands of timber with $i+\phi$. This correction coefficient is accurate if there will be perpetual static-state rotation cycles. This condition is violated for nonlinear problems addressed by ECHO. For these nonlinear problems, the correction coefficient will not be constant over time. For most applications when land holding costs are thought to be significant, however, the correction coefficient can be assumed to be a constant and then varied parametrically to find a constant correction coefficient that maximizes the present value of the results. This procedure is very simple, and for all practical applications appears to be more than adequate.

McDonough and Park (1975) have reformulated the problem solved by Walker (1971) as an optimal control problem utilizing the discrete maximum principle. They changed the time intervals from one to ten years, aggregated the initial inventory data into ten-year age classes, and increased the regeneration period from one to ten years. When ECHO is used with these data changes it gives almost identical results with the optimal control solution. The initial harvest levels, the steady-state equilibrium harvest levels, and the present values all differ by less than one percent. This good agreement between two very different solution techniques serves to validate both approaches.

Intrilligator (1971) has shown how the maximum principle can be considered a dynamic generalization of the Lagrange multipliers of the Kuhn-Tucker conditions for static optimization problems. The relationships between the maximum principle and dynamic programming are discussed by several writers such as Leitmann (1962), Aris (1963), and Fan and Wang (1964) who show that one approach can be derived from the other. This obviously must be true or otherwise one of them would not be valid. However, the way problems are approached by each method is quite different in spite of the common thread that ties them together. Despite the somewhat ad hoc nature of the ECHO solution technique which follows, there is a substantial body of economic and mathematical optimization theory that provides strong analytical support.

It is important to note that for forest resource planning and policy formation, the ultimate objective is not the precise formulation of abstract mathematical models, nor the mathematical optimization of these models by using one or more of the alternative methods that may be available. The ultimate objective is the numerical solution of the optimal policy which can be used. By utilizing the mathematical structure of a forest biological model and the economic conditions necessary for maximizing present value, the ECHO solution technique can help achieve this objective for a broad range of situations and empirical data. Only the basic concepts and applications are explained in the balance of this paper.

THE ECHO SOLUTION TECHNIQUE

The ECHO solution technique uses equation (I) to determine the optimal time path to a long-run equilibrium condition whatever the initial condition of the forest or the structure of the markets. This solution technique employs a concept that is identical to Bellman's Principle of Optimality (Bellman 1957). This Principle states that: "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision." When this Principle, which is really the essence of equation (I), is used to calculate harvests over time, given any initial harvest, W_o, the author has found through trial and error that the present value of all future net cash flows is a unimodal function of the initial harvest as illustrated by Figure I. The solution technique is to arbitrarily pick a trial initial harvest level and then use equation (I) and the demand or marginal revenue equations to calculate subsequent harvests. This is done simply by calculating the harvest levels forward through time which are required to maintain the equality condition expressed by equation (I). This process is continued until one of three rejection criteria are met which indicate the trial harvests are not optimal, but too high or too low. These rejection criteria are:

1) The harvest level needed to satisfy equation (I) becomes negative. This indicates that the initial harvest level was set too low.

2) The value $MR_t - MC_t$ becomes negative. This indicates that the initial harvest level was set too high.

3) All the forest inventory becomes exhausted and there is insufficient volume to satisfy the harvest calculated. This indicates that the initial harvest level was set too high.

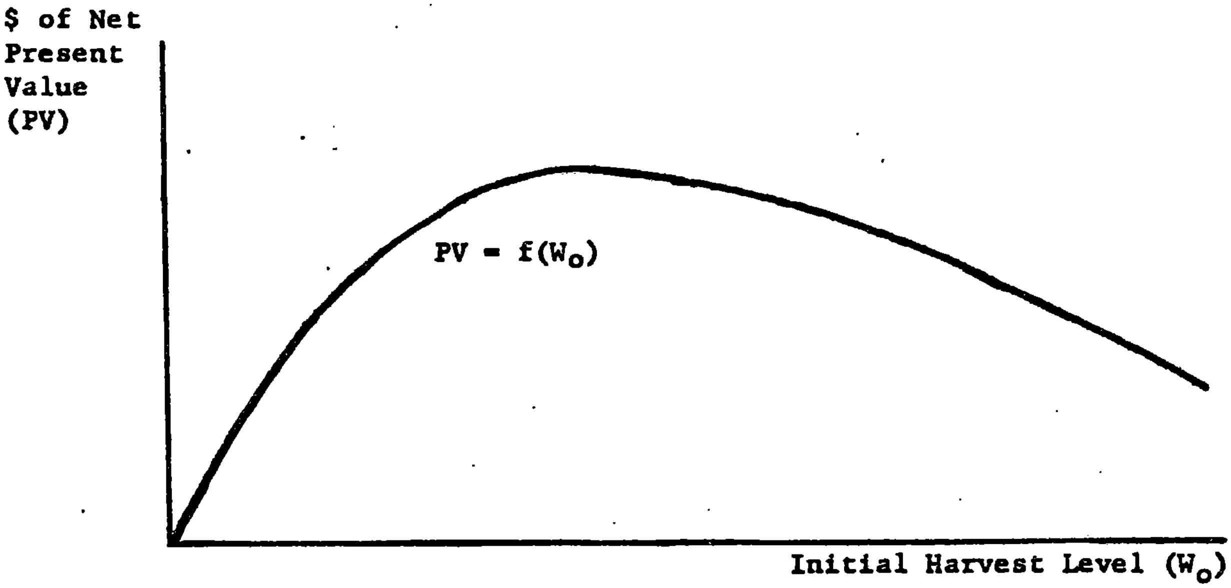


Figure 1. Present value as a unimodal function of the initial harvest decision followed by harvests calculated using equation (I).

The initial harvest level is then adjusted up or down by some specified amount and the whole calculation procedure starts again with this new harvest level. This process continues until trial initial harvest levels are obtained which bracket the optimum harvest level. The next trial harvest level is then set midway between these two levels and the iterations continue, always setting successive trial harvest levels midway between the last two which bracket the optimum level. These trial harvest levels then converge on the optimum level. The calculations are stopped when the difference between two levels which bracket the optimum level becomes sufficiently small that continued iterations are unwarranted. This process is therefore called an iterative optimum-seeking binary search technique.

ECHO solutions are always approximations, but the numerical results can be made as accurate as the user desires. The difference between two initial harvest levels which bracket the true optimum can be made as small as the precision of the computer permits and still cause the iterations based on these trial initial harvests to lead to rejection criteria indicating one set of harvests is too high and the other too low. The reason for this is that the true optimum is to just exhaust a particular age class at some point in time. The harvest levels that are too low leave part of this particular age class to grow another time period and provide V_{t+1} for use in equation (I). The harvest levels that are too high more than exhaust this age class at the optimal point in time and use the next age class in the array of harvest priorites for both V_t and V_{t+1}. Up until this point in time, t, the results of the two iterations are almost identical and can be considered optimum. Since the inventories at the next point in time, t+1, are also practically identical, this next point in time is treated as though it were a new initial point, and a new set of iterative calculations are begun. This process can be continued indefinitely. The user can either stop the calculations at a specified time horizon or can have them stop automatically when a long-run steady-state equilibrium is obtained.

If the demand curves are not changed over time, or if they change at some constant rate, the harvests will eventually reach a long-run equilibrium. This equilibrium may closely approximate a perfectly regulated forest condition, however harvests will cycle or fluctuate about the constant harvest level of a regulated forest. This is due to the nature of difference equations. When cycling occurs, the model will be harvesting all timber in the oldest age class. The length of the harvest cycle will be the same as the number of age classes in the long-run steady-state equilibrium forest. Figure 2 shows the relationship between the long-run equilibrium age class distribution and the harvest cycle.

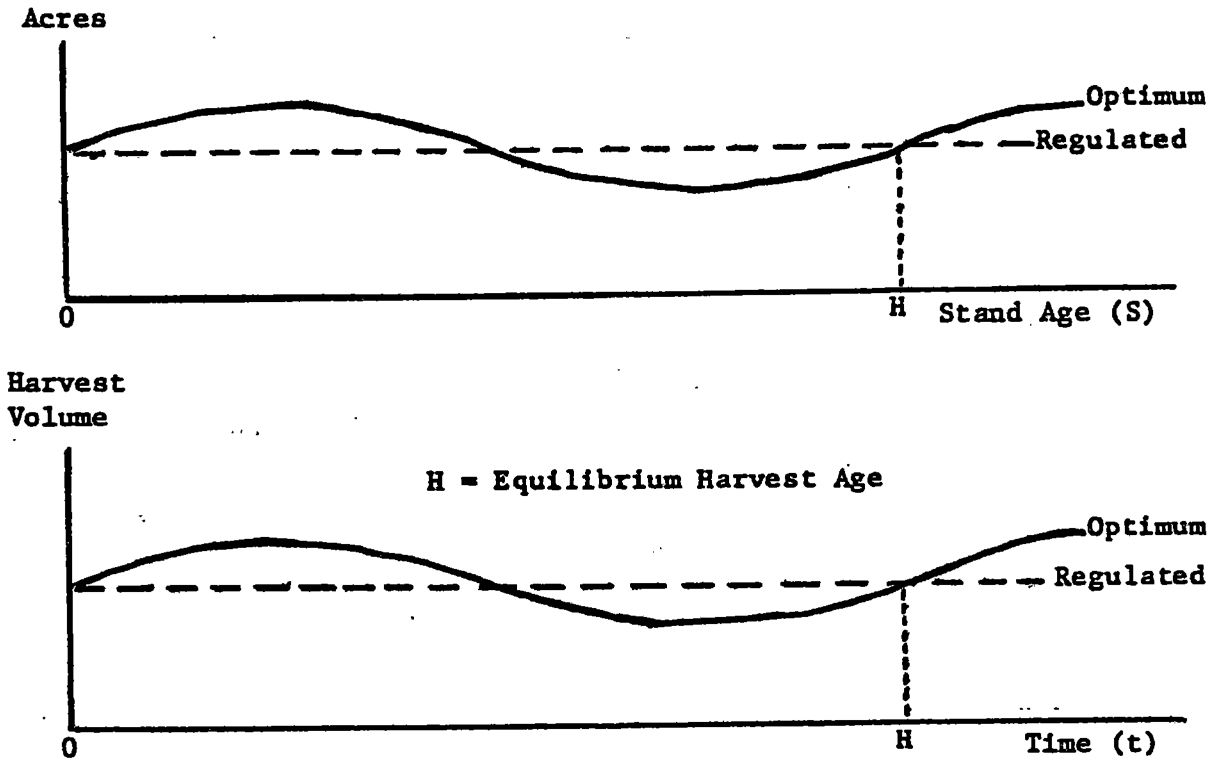


Figure 2. The relationship of age class distribution and harvest cycles in an optimum long-run equilibrium forest and a perfectly regulated forest.

The harvest volume for each point in time will be determined by the number of acres that have reached equilibrium harvest age and the volume per acre. The higher the interest rate used, the lower this equilibrium harvest age. Decreasing the amplitude of the cycles would involve harvesting some timber earlier or later than the equilibrium harvest age. The oscillating equilibrium is obtained when the present value benefits from decreasing the amplitude of the cycles are just offset by the costs of harvesting non-equilibrium age timber. This oscillation is caused by the discrete nature of difference equations. The longer the time period used for a given problem, the greater the amplitude of the steady-state cycles. If time were treated as continuous rather than discrete, the cycle amplitude would be zero.

This binary search solution technique is not an analytical technique. But as Chiang noted (1967) in his description of an iterative method for solving first-order difference equations, this technique is immensely revealing of the properties of an analytical solution. It points out clearly the manner in which a time path is generated. In general, the result for any particular point in time, W_t, will always depend in a specified way on the result of the immediately preceding point in time, W_{t-1}. Thus any given initial value, W_o, will successively lead to W_1, W_2, W_3, ... etc., via the prescribed pattern of change. For $W_o < W_o'$ the prescribed pattern initiated by W_o will always lie below the pattern initiated by W_o'.

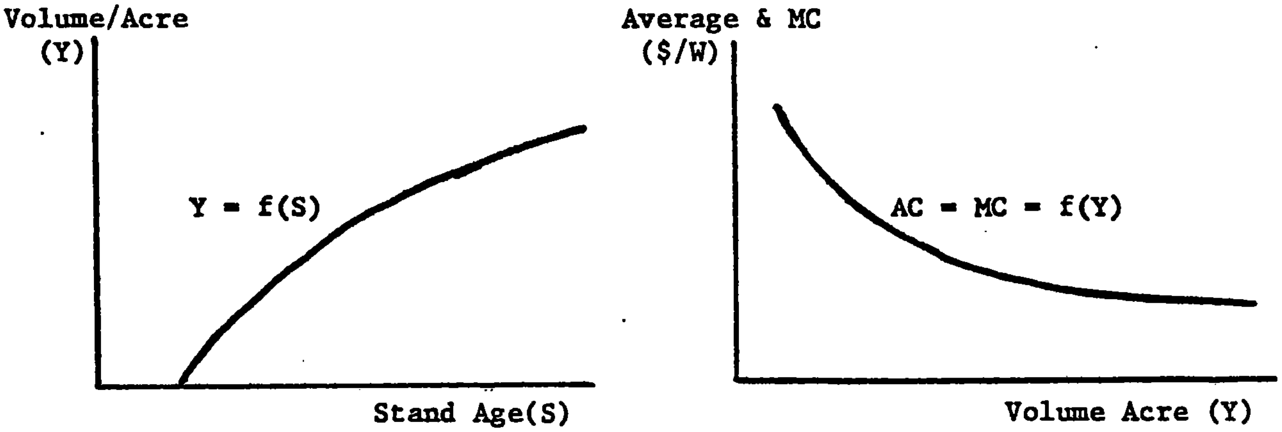
Once the optimal harvest level for any point in time is known, the relationship expressed by equation (I) determines the optimal harvest levels for all points in time. For example, if the optimal harvest level for time t=0 is known, the initial forest inventory and the selected harvest priorities (i.e. oldest first) will determine the age class of the last unit of wood harvested, hence $V_{t=o}$. The acres harvested at time t will support the youngest age class in the forest at time t+1. The actual value assigned this age class is dependent upon the regeneration period. If the time between t and t+1 is one year, and the regeneration period is one year, the acres harvested at time t will have a zero age stand on them at time t+1. Other acres in the inventory are adjusted upwards by one age class and the values for V_{t+1} and MC_{t+1} are determined. The value for $V_t(MR_t - MC_t)(1+i)$ is then calculated. It is then possible to solve for MR_{t+1} using:

$$MR_{t+1} = \frac{V_t(MR_t - MC_t)(1+i)}{V_{t+1}} + MC_{t+1} \qquad \text{(VII)}$$

Once MR_{t+1} is calculated, it is used to solve for the required harvest volume, W_{t+1}, to achieve this marginal revenue using the total marginal revenue equation of the form $W_{t+1} = f(MR_{t+1})$. This process is continued for subsequent time periods using the difference equation relationships to select harvest volumes which achieve necessary conditions for an optimal solution.

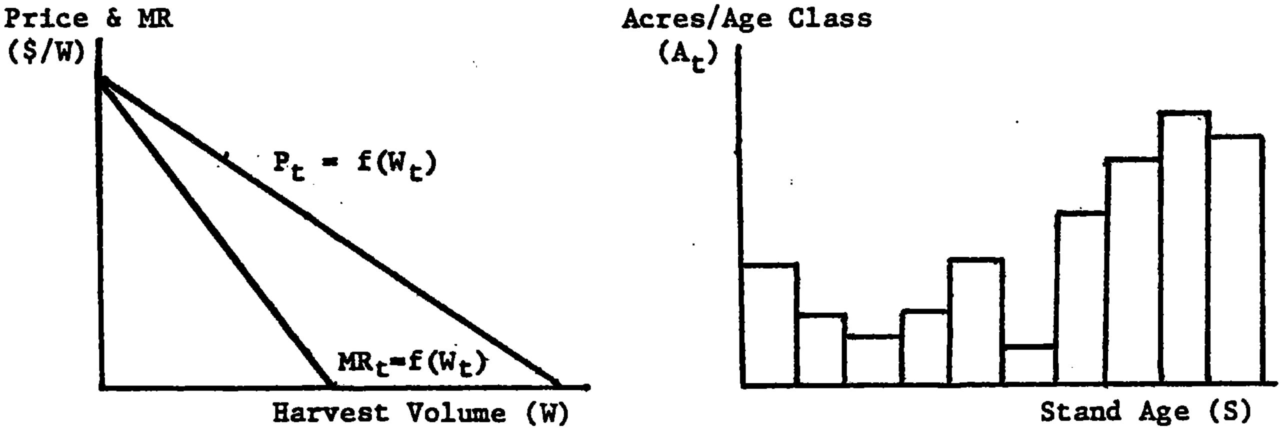
Even the mathematically unsophisticated can gain great insight into the nature of the solution by taking pencil in hand for a numerical example and manually working through a few iterations. The empirical data provided by Walker (1971) can be used as a guide. Only a faint recollection of high school algebra is required. An understanding of derivatives, Lagrange multipliers, gradient vectors, and other paraphernalia of advanced mathematics and optimization theory is not essential. The foregoing discussion was designed to point the interested reader to relevant mathematics literature. Relevant economics literature has already been cited (Walker 1971) with the significant exceptions of Dorfman (1969) and Gordon (1967).

Selected ECHO input data used by Walker (1971) in a slightly abridged form is shown in Figure 3. This data, plus specific forest management costs and an interest rate, i, are all that are needed to use ECHO. A correction coefficient, Φ, which was discussed earlier, can also be used if the user wants to account for land holding costs. The author has found, however, that this correction factor has not significantly affected the results of any applications made to date with ECHO using realistic rates of interest.



Stand Yield Equation
(a)

Logging Cost Equation
(b)



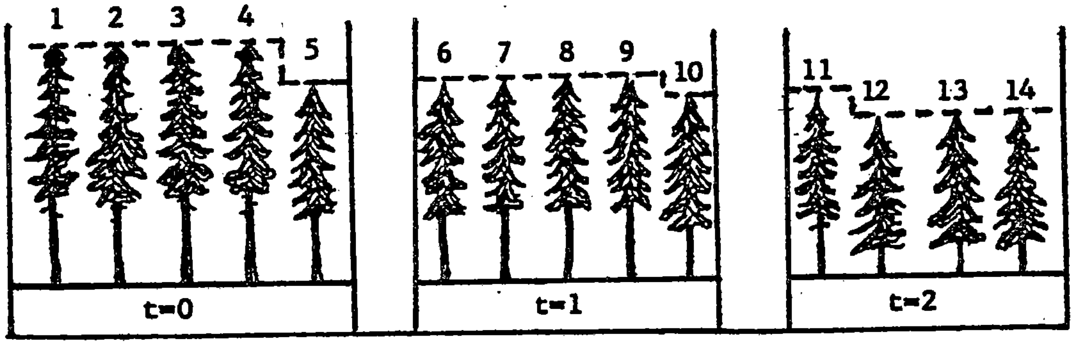
Demand & Marginal Revenue
(c)

Age Class Distribution
(d)

Figure 3. Selected ECHO Input Data

It may be useful at this time to review the way equation (I) is used with this type of data. The demand and marginal revenue data are the most foreign to forest management planners, but are key elements in the use of ECHO. If a demand curve is a straight line such as the one shown in Figure 3c, the associated marginal revenue curve will start at the same point on the vertical axis but be twice as steep. Marginal revenue is the change in total revenue when there is a one unit change in the quantity offered for sale. This is a very important concept in understanding how ECHO operates. When you change the quantity, you also change the price that is used to calculate total revenue. Marginal revenue curves are just a short-cut way of measuring the change in total revenue. In practice, the demand and marginal revenue curves may be more complicated than this. They may not be straight lines, but may have kinks or discontinuities at points where added volume means adding a shift to a mill or going onto overtime.

Now let's apply this to a forest. Figure 4 shows a highly simplified schematic view of a forest inventory with "trees" numbered in terms of their logging priority. These "trees" may in fact represent individual trees, stands of even-aged trees, or portions of uneven aged stands that will be partially cut. For application where many stands are of uneven age, "trees" 3, 8, and 13, for example, could represent components of the same stand that will be logged at different points in time. For simplicity, let's assume we are dealing with individual trees. Note that trees 1-4, 5-9, 10-11 and 12-14 are drawn to indicate that they are in the same age class. The separate grouping of trees 1-5, 6-10, and 11-14 indicate the harvests at different points in time. Tree 5, for example, is the last tree harvested at time zero, the time of decision-making, and tree 6 is the first tree harvested at time one, or one year hence. Figure 4 only shows the highest priority trees for harvesting at times zero, one, and two, with the balance of the forest off the page to the right. Whenever the highest priority trees on the left are harvested, the bare land moves right to the back of the line for regeneration.



$$V_t(MR_t - MC_t) = \frac{V_{t+1}(MR_{t+1} - MC_{t+1})}{(1+i)} \qquad (I)$$

Figure 4. Optimal Harvest Calculation

Equation (I) has been included in Figure 4 to help focus on the mathematical relationship that exists between the last tree harvested at each point in time and the first tree harvested at the next point in time.

For this schematic example, the equation must apply between trees 5 and 6, 10 and 11, 14 and 15, etc., through all future time periods. It may be useful for the reader to refer again to Figure 3 and to the definitions of the variables used in equation (I). Now assume that the equality expressed by equation (I) exists between tree 5 harvested at time zero and tree 6 harvested at time one. The harvest plan could be incrementally changed by either shifting tree 5 back to time one or tree 6 ahead to time zero. This, however, would violate the equality conditions of equation (I). The sum of the present value of the net revenues of both time zero and time one would no longer be maximized. Shifting tree 5 to time one would reduce the total net revenue at time zero by more than it would increase the present value of the total net revenue at time one. Likewise, shifting tree 6 back to time zero would decrease the present value of the total net revenue at time one by more than it would increase the total net revenue at time zero. The reader can apply the same logic to the relationship between trees 10 and 11 for times one and two.

RESULTS AND POLICY IMPLICATIONS

The results of an ECHO solution using empirical data are discussed in detail by Walker (1971) and need not be repeated in this paper. Figure 5, however, shows how the ECHO model can be used to demonstrate the economically irrational implications of even-flow policies which are very prevalent among forest management agencies.

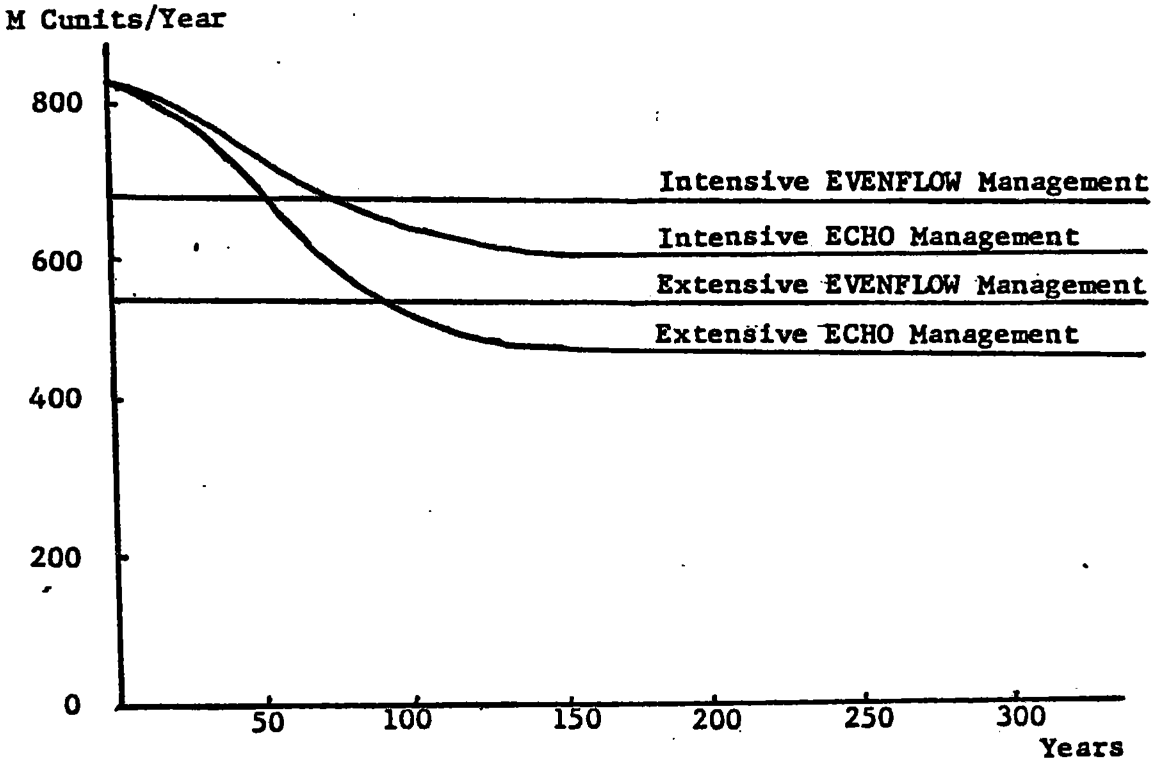


FIGURE 5. ECHO and even-flow harvest plans

The two horizontal lines in Figure 5 are for two even-flow management plans based on the data provided by Walker (1971). The lower horizontal line labeled Extensive EVENFLOW Management is based on the rotation age that maximizes the mean annual increment of the yield equation used by Walker (1971). The upper horizontal line labeled Intensive EVENFLOW Management uses the same approach assuming the so-called extensive management yields can be increased by twenty-five percent. The harvest in both these even-flow plans is constrained to the long-run sustained-yield level.

The two ECHO plans, however, are based on no even-flow constraint, but maximize present value based on market and physical production opportunities. The harvest schedule labeled Extensive ECHO Management is taken directly from the results provided by Walker (1971). The Intensive ECHO Management schedule has the extensive management yields increased by twenty-five percent. All the other input data for both the ECHO and EVENFLOW plans are identical to those used by Walker (1971).

Table 1 shows the differences in the present values of the management plans corresponding to the harvest schedules shown in Figure 5. These present values are all based on a six percent rate of interest.

Table 1

6% Present Value of Management Plans
($ Millions)

	Extensive Management	Intensive Management	Difference
EVENFLOW Plans	452.3	524.9	72.6
ECHO Plans	606.0	607.7	1.7

The EVENFLOW plans have substantially less present value than the ECHO plans for the same level of management. This suggests the magnitude of the opportunity costs of following an even-flow policy. This should not surprise anybody, however, since the proponents of even-flow make few pretenses that it is economically justifiable. It has become increasingly fashionable, however, to use a so-called "allowable cut effect" to justify intensive forest management practices. In this example, the "allowable cut effect" would indicate that the present value of the costs that could be incurred economically to increase yields by twenty-five percent is $72,600,000. The actual value for this increased yield is only $1,700,000. A lengthy discussion of the irrationality of an "allowable cut effect" is provided elsewhere by Walker (1971, 1974 and 1975) and need not be repeated here. Systems analysts should take note, however, that they must carefully consider the significance and relevance of the harvest flow constraints commonly employed in more conventional optimization techniques or they will unwittingly continue to present erroneous conclusions.

FUTURE WORK

ECHO is currently being used for a research project at the University of Washington to conduct an economic assessment of the United States timber situation. ECHO is being modified to handle spatial as well as intertemporal optimization. This expanded use of ECHO will divide the United States into several timber supply and demand regions much along the lines used by Holley and others (1975) in an Interregional Timber Model (ITM). This marriage of the ECHO and ITM concepts will provide a means of simulating the use of the market mechanism to guide forest management decisions.

A brief explanation of how ECHO and ITM concepts can be combined is illustrated by Figure 6. Figure 6 is analogous to Figure 1 except it is drawn for two rather than one timber supply region or forest area, and two or more timber demand regions or markets that are spatially separated. The straight forty-five degree lines in Figure 6 are total initial harvest lines with $W^1_{T_o} < W^2_{T_o} < W^3_{T_o}$ ....etc. Each point along a given line has the same total harvest, but consists of different proportions of this total harvest coming from each supply region. Along the Y axis the total harvest comes solely from supply region Q. Along the X axis the total harvest comes solely from supply region R. A specified initial harvest in each of Q and R will designate a point on one of the total harvest lines.

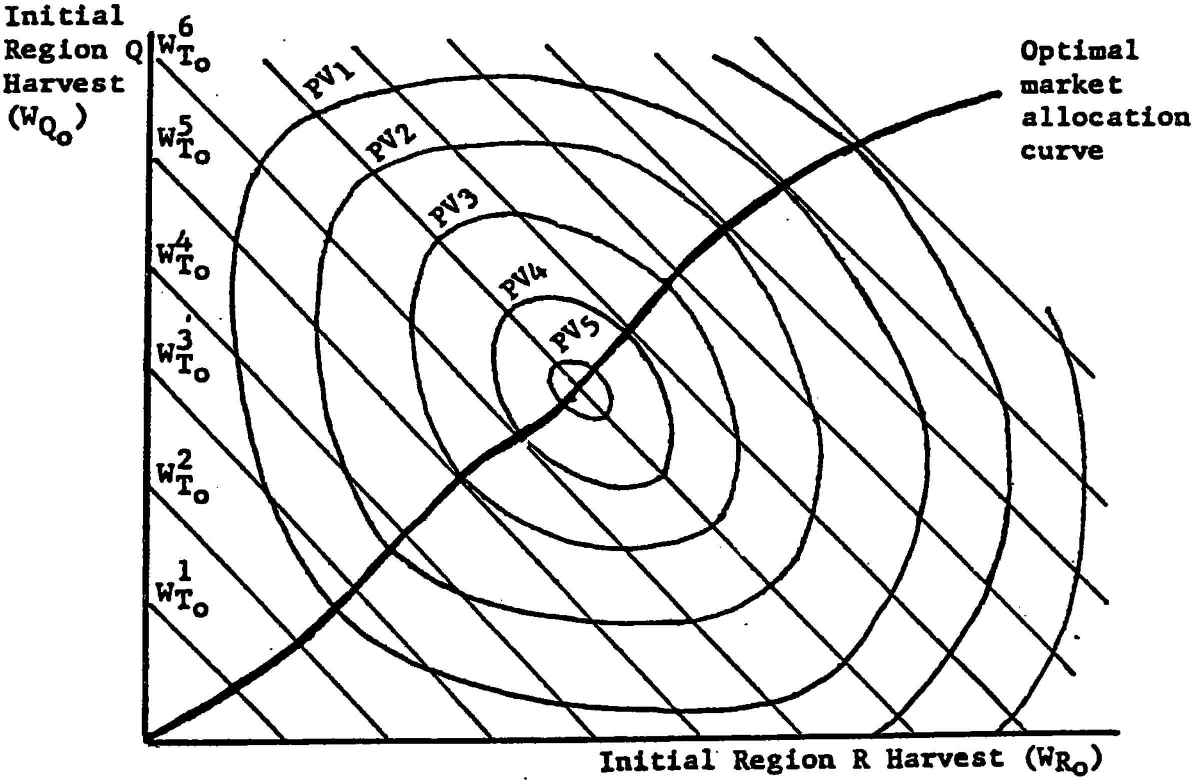


Figure 6. Present value as a unimodal function of the initial harvest decision followed by harvests calculated by using equation (I) for two timber supply regions Q and R.

To achieve a point on the present value surface designated by the contours such that $PV_1 < PV_2 < PV_3$ ....etc., requires a profit maximizing allocation of the timber to the various markets for each possible set of harvests from each of the supply regions. With only two markets this allocation can be solved algebraically using the conditions that the net marginal revenue must be equal for the timber allocated from each supply region to each demand region. For more than two demand or supply regions the ITM allocation model may be useful, especially if it can be successfully developed by Haynes (1974) to adequately handle negative sloped timber demand curves in each region. If not, an additional iterative solution technique for this allocation process can be added to ECHO or an optimal control algorithm can be used depending upon their relative efficiency.

Once the present value surface has been reached, the present value can be increased without changing the total harvest by moving along a total initial harvest line to where it crosses the "optimal market allocation curve." This optimal market allocation curve traces out the profit maximizing combinations of harvest in each of the two supply regions for all possible levels of total initial harvest, and reduces Figure 6 to the equivalent of Figure 1. Along the optimal market allocation curve, total present value becomes a unimodal function of just the total initial harvests from all possible supply regions. Once this function is defined, the ECHO solution technique will calculate the optimal levels of harvest over time for each timber supply region.

ECHO is in the process of being expanded along the lines briefly mentioned above to conduct an economic assessment of the United States timber situation. It is expected this work will be completed by the end of 1975.

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