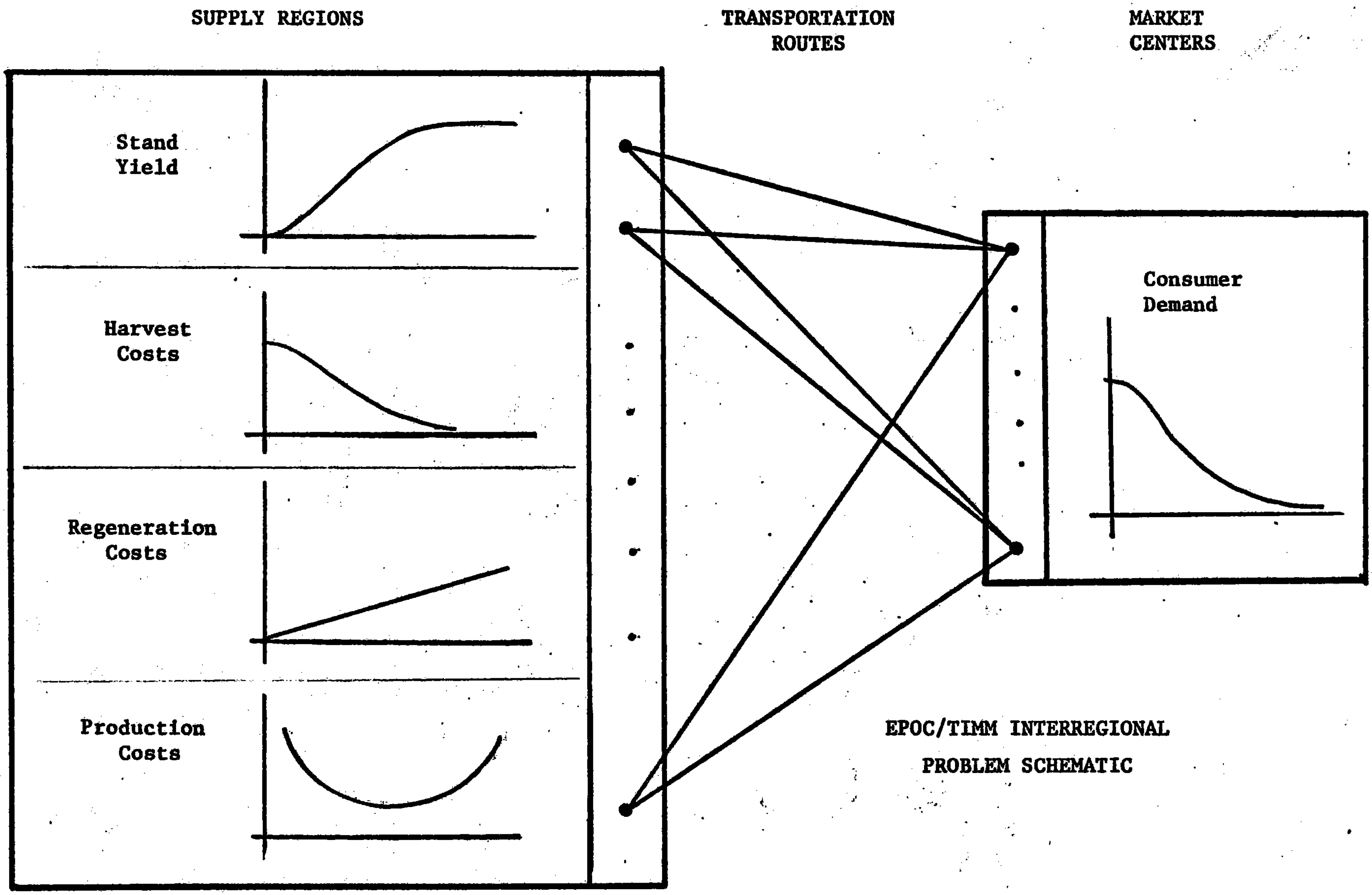


EPOC/TIMM INTERREGIONAL PROBLEM SCHEMATIC

Assumptions on basic equations

1) In NPV it is assumed all timber harvested in period n is sold in period n; i.e. no inventories accumulated. $(G \equiv 0)$.

2) In forest state functions assume age class span is same as economic planning period length.

Also, note that more general inequality constraints may be imposed.

DISCRETE OPTIMAL CONTROL PROBLEM	EPOC/TIMM EQUATIONS
Optimize $$P(X,U;c) = \sum_{n=1}^{N} P_n(X_{n-1},U_n;c_n) + G(X_N)$$	$$NPV = \sum_{n=1}^{N} \frac{R_n - C_n}{(1 + r_e)^n}$$
subject to $$X_n = F_n(X_{n-1},U_n;c_n) \quad \text{with } X_0 \text{ given;}$$	$$x_{1,j,n} = x_{1,j,n-1}(1 - u_{1,j,n}) + \sum_{k=2}^{I_j} x_{k,j,n-1}u_{k,j,n}$$ $$x_{2,j,n} = x_{1,j,n-1}u_{1,j,n}$$ $$x_{i,j,n} = x_{i-1,j,n-1}(1 - u_{i-1,j,n}) \quad i = 3,\ldots,I_j$$
$$U_{n,min} \le U_n \le U_{n,max}$$	$$0 \le u_{i,j,n,min} \le u_{i,j,n} \le u_{i,j,n,max} \le 1$$

Note that discrete maximum principle leads to periodwise decomposition of sequential problems.

that it extends the classical calculus of variations

that the λ_n's are similar to Lagrange multipliers, and to the multipliers determined by the "master planner" in Dantzig-Wolfe decomposition.

DISCRETE MAXIMUM PRINCIPLE

Necessary conditions for an optimum:

1. $\frac{\partial H_n}{\partial U_n} = 0$, or $Z_n(X_{n-1}, U_n)$ attain boundary values.

2. There exist Lagrange multipliers λ_n such that

$$\lambda_n = \frac{\partial H_{n+1}}{\partial X_n}$$

3. $X_n = \frac{\partial H_n}{\partial \lambda_n}$

where

$$H_n(X_{n-1},U_n) = P_n(X_{n-1},U_n) + \lambda_n F_n(X_{n-1},U_n)$$

SOLUTION ALGORITHM

1. Input initial guesses for control variable (management policy) values, and known initial state (forest inventory)

2. Calculate inventories, revenues, and costs for all periods in planning horizon.

3. Recursively evaluate λ_n and H_n, starting with $\lambda_N = \partial G/\partial X_N$,

$$H_N = P_N + \lambda_N F_N, \quad \lambda_{N-1} = \lambda H_N/\partial X_{N-1}, \text{ etc.}$$

4. Update control variable values using

$$U_n^{(m)} = U_n^{(m-1)} + \varepsilon \frac{\partial H_n}{\partial U_n}^{(m-1)}$$

5. Check

$$U_{n,min} \leq U_n^{(m)} \leq U_{n,max} .$$

If not satisfied, reduce ε; and return to 4.

6. Test convergence criteria. If satisfied, Stop. If not, return to 2, with updated control values.

Assume each supply region corresponds to a type site.

$\Rightarrow$ $j = \sigma$ and $J = S$.

Assume all products manufactured from raw product k can be characterized by $\bar{\xi}_k$

REVENUE CALCULATIONS

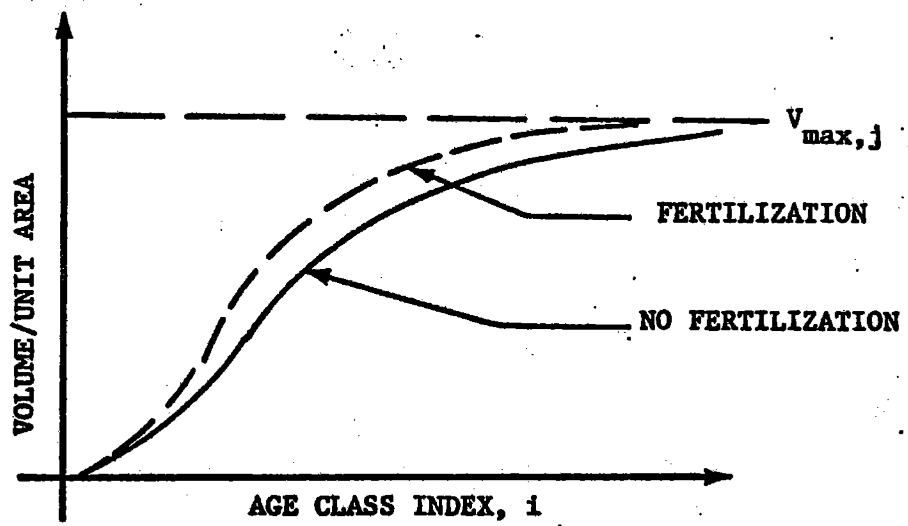
$$v_{i,j} = v_{max,j} \exp[-\alpha_j / g(w,i)]$$

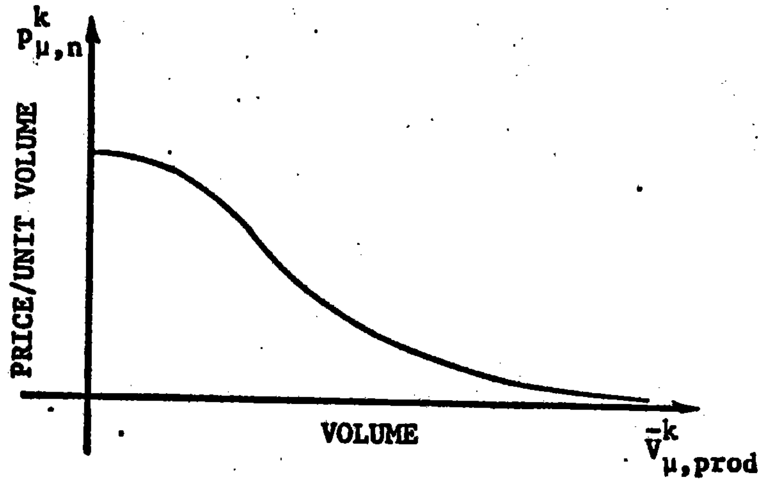
$$V^k_{\mu,n} = \sum_{\sigma=1}^{S} \sum_{i=I_j^{k-1}+1}^{I_j^k} c^k_{\sigma\mu,n} x_{i,j,n-1} u_{i,j,n} v_{i,j}$$

$$\bar{V}^k_{\mu,prod} = \bar{f}_k(V^k_{\mu,n})$$

$$p^k_{\mu,n} = p^k_{\mu,max} \exp[-B^k_\mu (\bar{V}^k_{\mu,prod})^2]$$

$$R_n = \sum_{k=1}^{K} \sum_{\mu=1}^{M_k} p^k_{\mu,n} \bar{f}_k(V^k_{\mu,n})$$

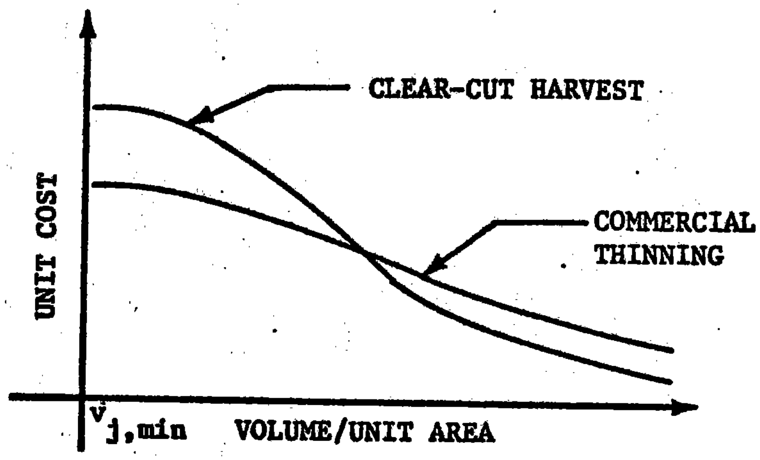




EVALUATION OF COST FUNCTIONS

$$C_{F,n} = \sum_{j=1}^{J} a_{j,n} A_{j,n,T}$$

$$C_{R,n} = \sum_{j=1}^{J} b_{j,n} x_{1,j,n-1} u_{1,j,n}$$



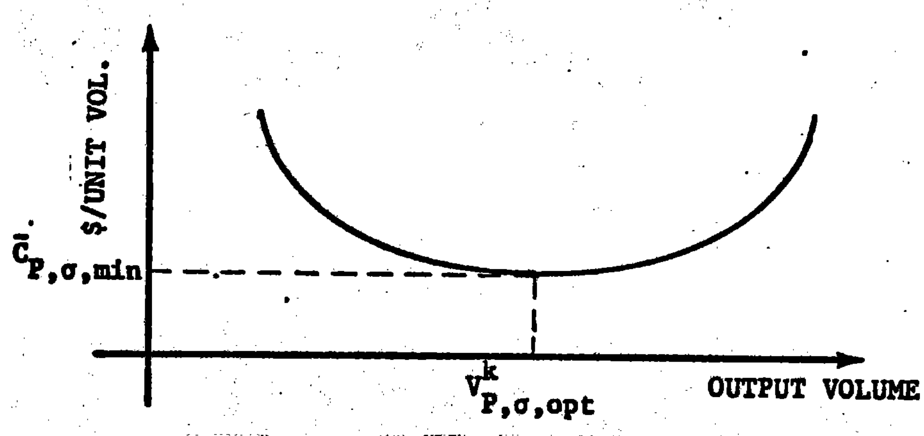
$$\bar{c}_{H,i,j,n} = \bar{c}^{0}_{H,i,j,n} \exp[-\chi_j (v_{j,min} - v_{i,j})^2]$$

$$A_{H,i,j,n} = x_{i,j,n-1} u_{i,j,n}$$

$$C_{H,n} = \sum_{j=1}^{J} \sum_{i=2}^{I_j} \bar{c}_{H,i,j,n} v_{i,j} A_{H,i,j,n}$$

COST FUNCTIONS (continued)

$$V^k_{\sigma,n} = \sum_{i=I_j^{k-1}+1}^{I_j^k} x_{i,j,n-1} u_{i,j,n} v_{i,j}$$



$$V^k_{P,\sigma,n} = \bar{f}_k(V^k_{\sigma,n})$$

$$\Delta V^k = \frac{V^k_{P,\sigma} - V^k_{P,\sigma,opt}}{V^k_{P,\sigma}\bar{C}^k_{P,\sigma,min}}$$

$$V^k_{\sigma\mu,n} = c^k_{\sigma\mu,n} V^k_{\sigma,n} \qquad V^k_{P,\sigma\mu} = \bar{f}_k(V^k_{\sigma\mu}) \qquad \rho_k = W_k/\bar{V}_k$$

$$\bar{C}_{P,\sigma} = \frac{V^k_{P,\sigma}\bar{C}^k_{P,\sigma,min}}{2}\left(\varepsilon^{\Delta V^k} + \varepsilon^{-\Delta V^k}\right)$$

$$c^k_{T,\sigma\mu} = \xi^k_{\sigma\mu} d_{\sigma\mu} V^k_{P,\sigma\mu}/\rho^a_k + \eta^k_{\sigma\mu} V^k_{P,\sigma\mu}/\rho^b_k$$

$$C_{P,n} = \sum_{\sigma=1}^{S} \sum_{k=1}^{K} \bar{C}_{P,\sigma}$$

$$C_{T,n} = \sum_{k=1}^{K} \sum_{\substack{\text{over } R_k \\ \text{routes}}} c^k_{T,\sigma\mu}$$

$$C_n = C_{F,n} + C_{R,n} + C_{H,n} + C_{P,n} + C_{T,n}$$

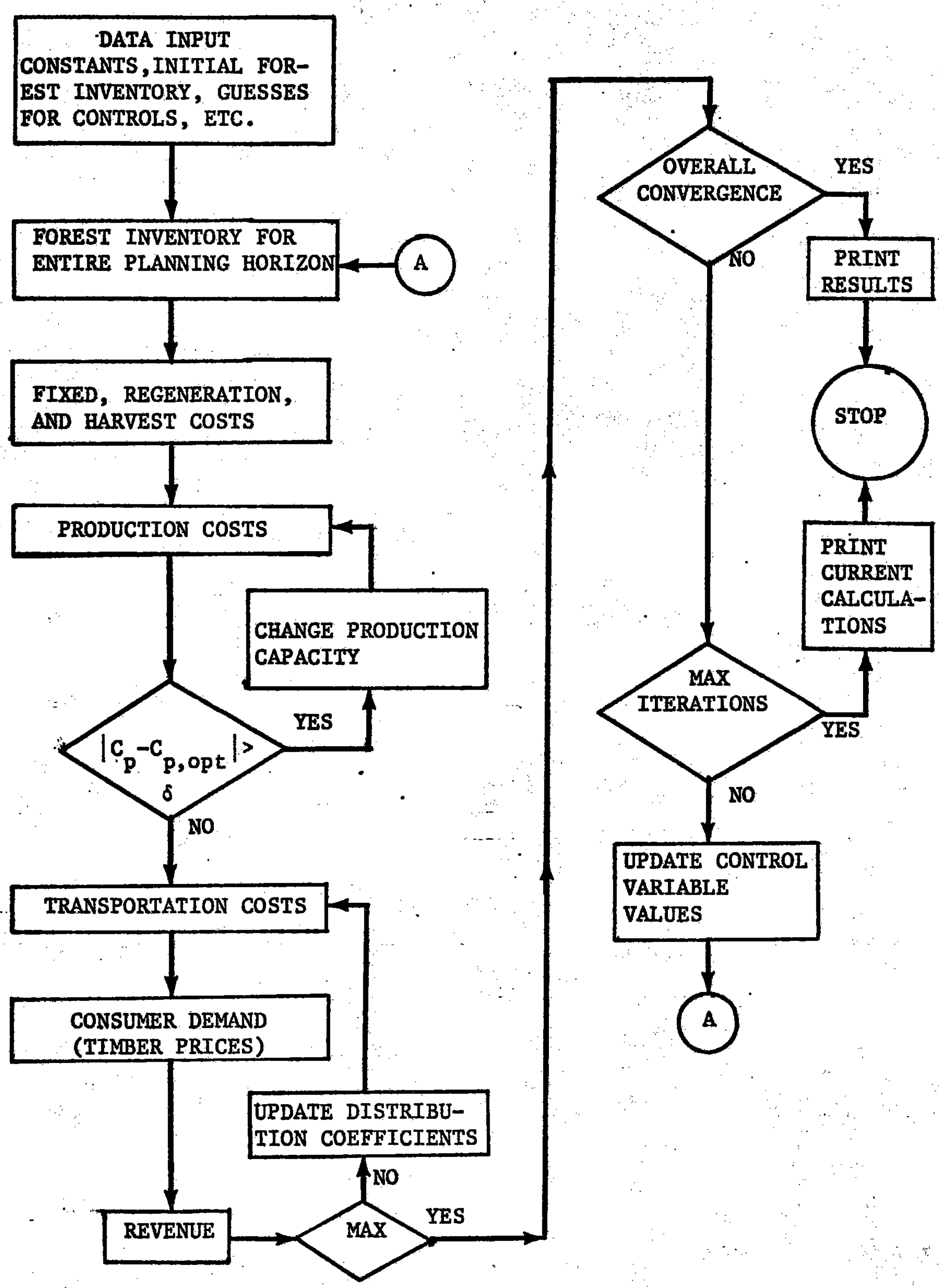


Figure 6.--EPOC/TIMM flow chart for interregional planning model, including production costs and capacity changes, and product distribution