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SUBJECT: Shuttle Manifest Mission Shaping Problem

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SUMMARY - The mission shaping problem for the Shuttle for a single launch has been defined in a simple way, for circular target orbits, by Escobal in Reference 1. The purpose of this memorandum is to describe a methodology that is fairly robust, to solve the non-linear optimization problem posed in Reference 1. This methodology uses the PROSE programming language (see Reference 2) to express the problem to the computer (the I.B.M. 370/3033 was used). The advantage of PROSE in the solution of non-linear optimization problems is that it has a powerful calculus capability and a set of general solvers (see Reference 3) that are easily adaptable to the mission shaping problem.

GENERAL DESCRIPTION - The object of the mission shaping is to maximize the performance functional

$$J = P - \sum_{i=1}^{n_p} W_i \quad , \qquad (1)$$

as defined in Reference 1, where n_p is the number of payloads in the mission.

J as formulated is a function of the following independent variables (variables to be optimally solved for), and other fixed parameters specified in Reference 1.

- r = Radius increment from minimum radius shuttle orbit,
- i_S = Inclination of shuttle orbit,
- Ω_{SO} = Initial longitude of shuttle orbit at jettison time,
- i_{Ti} = Inclination of each transfer orbit ($i = 1, ---, n_p$),
- $\Delta\Omega_{T,Fi}$ = Impulsive nodal change between transfer and each final orbit ($i = 1, ---, n_p$),

λ_i = Transfer revolution multipler for each payload (i = 1,---, n_p),

τ_i = Dwell time inequality dummy (slack) variable for each payload (i = 1, ---, n_p).

The solver in PROSE that was used to maximize J in (1) was JUPITER, which is a first order gradient type method (see Reference 3 for more details about JUPITER).

To facilitate the solution process it was found useful to transform (1) to

$$J^* = f_s \ln (J_{max} - J)^2, \tag{2}$$

and minimize J^*.

Convergence using (2) was faster, more stable, and more accurate than if (1) was solved directly. It was simply required to determine f_s and J_{max} a priori to the optimization process. It turned out that f_s = 50 and J_{max} = 13000 kg were appropriate values for the cases considered here. These values were easily determined by a couple of trial runs of the computer program. They should be appropriate for a wide spectrum of cases.

<u>TWO MISSION SHAPING CASES</u> - The two mission shaping cases considered here had n_p = 1 and n_p = 2 respectively. All the fixed parameters had the values specified by Escobal in Reference 1 except Ω_{F2} = 340 deg, T^* = 150 days, and H^* = 259.28 km instead of the ones specified. These changes would make little or no difference in the solution process.

As in all optimization problems the selection of starting guesses for the independent variables is very important. For instance bad choices can cause slow convergence or even divergence of the solution process. It was determined here that certain independent variables needed more care in starting guess selection than others, in that they effected the solution process greatly. These variables were found to be i_S and i_{Ti}. Thus, in order to make the solution methodology as robust as possible it was designed to be as independent as possible of user selections of starting guesses for these variables. Therefore, only an interval of values for i_S need be specified by the user of the methodology. The methodology uses this interval and certain other general information to start the solution process. The solutions for the two cases mentioned above are shown together with their starting guesses, in Tables 1 and 2 respectively. Both cases were run with the convergence tolerance in JUPITER set to 10^{-6} (see Reference 3 for details). Note that the starting guess for i_{Ti} is determined from that value arrived at for i_S by the relationship

$$i_{Ti} = i_S + (i_{Fi} - i_S)/2,$$

as specified in Reference 1 (also, the i_{Fi} values are defined in Reference 1 as fixed parameters). The results in Tables 1 and 2 show the starting guesses for i_S are required to be known to the nearest radian. Contrast this with the requirement that i_S be known to the nearest degree in Reference 4!

<u>CONCLUSIONS</u> - While the methodology described here is fairly robust and complete as far as it goes, much work still remains to be done. The following items are obviously important.

1. Test the optimization on cases with more payloads, i.e., $n_p = 3$ and $n_p = 4$.
2. Extend the methodology to elliptical orbits as specified by Escobal in Reference 5.
3. Extend the methodology to include more and different independent variables in the optimal solution.
4. Evaluate other solvers as they become available in PROSE, e.g., solver GRG (See Reference 6).

REFERENCES

1. P. R. Escobal, "Two Impulse Optimum Shuttle Manifest Analyzer", Aerospace Memorandum, 11 August 1981.

2. Cybernet Services, "Prose - Procedure Manual", Publication Number 84003000, Revision B, Control Data Corporation, 1 January 1977.

3. Cybernet Services, "Prose - Calculus Operations Manual", Publication Number 84003200, Revision B, Control Data Corporation, 1 January 1977.

4. P. R. Escobol, "Progress Report: Shuttle Manifest", Aerospace Memorandum, 16 December 1981.

5. P. R. Escobol, "Extension of the Two-Impulse Optimum Manifest Problem to Permit Analysis of Elliptical Target Orbits". Aerospace Memorandum, 15 September 1981.

6. J. B. Mantell and L. S. Lasdon, "A GRG Algorithm for Econometric Control Problems", Annals of Economic and Social Measurement, 5 June 1978.

CASE $n_p = 1$

Variable Name (symbol)	Starting Guess (value)	Converged Value
r km	$\doteq 0$	$\doteq 0$
i_S radians	(1, 2)	1.387740
Ω_{SO} radians	.1745329 (10 degrees)	.1741926
i_{T1} radians	determined from i_S	1.438128
$\Delta\Omega_{T,F1}$ radians	$\doteq 0$	$\doteq 0$
λ_1 dimensionless	1	1
τ_1 minutes	464.7580	464.7580
J^* dimensionless	--------	$5.461940\text{x}10^2$
J kg	--------	$1.276445\text{x}10^4$

Table 1

CASE $n_p = 2$

Variable Name (symbol)	Starting Guess (value)	Converged Value
r km	$\doteq 0$	$\doteq 0$
i_S radians	(1, 2)	1.523392
Ω_{SO} radians	.1745329 (10 degrees)	.7247212
i_{T1} radians	determined from i_S	1.573886
i_{T2} radians	determined from i_S	1.573886
$\Delta\Omega_{T,F1}$ radians	$\doteq 0$	$\doteq 0$
$\Delta\Omega_{T,F2}$ radians	$\doteq 0$	$\doteq 0$
λ_1 dimensionless	1	1
λ_2 dimensionless	1	1
τ_1 minutes	464.7580	324.5012
τ_2 minutes	$\doteq 0$	$\doteq 0$
J^* dimensionless	-----------	7.946885×10^2
J kg	-----------	1.017330×10^4

Table 2

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